MIXED CONVECTIVE FLOW ON A HEATED SURFACE WITH INJECTION

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Mixed convection on a heated horizontal surface in the laminar flow regime has been investigated previously [1-3]. Martynenko and Sokovishin [1] have derived equations in the Boussinesq approximation for the laminar boundary layer in mixed convection and have obtained exact self-similar and approximate solutions of these equations by an approach similar to the Von Kármán–Pohlhausen method. Pirozhenko [2] has lifted the restrictions imposed by the Boussinesq approximation in the description of mixed convection and has used the Von Kármán–Pohlhausen method to determine the thicknesses of the dynamic, thermal, and diffusion boundary layers. Chen and others [3] have numerically solved the boundary-layer equations in the Boussinesq form and analyzed the mutual influence of forced and free convection in flow over a heated horizontal plate.

We now propose a formulation of the mixed convection problem for the cases of stable and neutral stratification of the medium [4] beyond the limits of the boundary layer.

It is established in a qualitative analysis of the problem that with a decrease in the temperature of the underlying surface the frictional stress is greatly reduced for both laminar and turbulent flow, and boundary-layer separation can take place. Asymptotic expressions for the frictional stress and heat flux are obtained by the method of Shvets [5]; for the laminar flow regime they agree with the results of the numerical calculations. An iteration-interpolation method [6, 7] and a digital computer are used to determine the limits of validity of the Boussinesq approximation and to show that under definite conditions free convection has little effect on the heat flux toward the flow surface.

1. Statement of the Problem. Let us consider a stratified gas flow on an arbitrary heated plane surface, through which is injected a heated gas of the same kind. This flow is described in the general case by the Navier–Stokes equations for laminar flow or by the Reynolds equations for turbulent flow [4]. In the case where the Reynolds number Re is large and the freestream mass flow rate is much greater than the injection mass flow rate $\rho_e u_e \gg (\rho v)_w$, the entire flow region can be partitioned into the boundary layer, in which molecular (molar) transport processes are significant, and the outer flow zone, in which viscous forces can be neglected. If it is assumed, in addition, that the composition of the gas remains unchanged, then the flow in the boundary layer is described by the system of equations

$$\frac{\partial \rho u}{\partial x}+\frac{\partial \rho v}{\partial y}=0; \tag{1.1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\partial p}{\partial x}-\rho g\sin\alpha; \tag{1.2}$$

$$\frac{\partial p}{\partial y}=-\rho g\cos\alpha,\ p=\frac{\rho RT}{M},\ \mu=\mu_{\text{м}}+\mu_{\text{т}},\ \lambda=\lambda_{\text{м}}+\lambda_{\text{т}}; \tag{1.3}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+k\left(u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}\right). \tag{1.4}$$

To close the system of equations (1.1)-(1.4) we assume that the turbulent kinematic viscosity ν_T in the inner and outer parts of the boundary layer is described by the Van Driest equations [8]

$$\nu_{\text{т}}=(0.4y)^2\left[1-\exp\left(-\frac{y}{A}\right)\right]^2\left|\frac{\partial u}{\partial y}\right|,\ A=26\nu_{\text{м}}\left(\frac{\tau_w}{\rho}\right)^{-1/2}; \tag{1.5}$$

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$$\nu_T = 0.0168\gamma_1 \left| \int_0^\delta (u_e - u)\, dy \right|, \quad \gamma_1 = \left| 1 + 5.5\left(\frac{y}{\delta}\right)^6 \right|^{-1}. \tag{1.6}$$

The influence of stratification inside the boundary layer is taken into account in the Van Driest equation (1.5) insofar as the density and molecular viscosity, which depend on the temperature, enter into the expression for A. It cannot be claimed, of course, that this treatment is absolutely correct. Unfortunately, in the literature we cannot find any data on the effects of stratification inside the boundary layer on the transport coefficients for turbulent flows. Here we need additional experimental data, the acquisition of which can be the object of an independent study. Consequently, the numerical results given below for turbulent flows can be regarded as predictions requiring experimental verification.

Equations (1.1)-(1.4) must be solved subject to the boundary conditions

$$u|_{y=0} = 0, \ (\rho v)|_{y=0} = (\rho v)_w(x), \ T|_{y=0} = T_w(x); \tag{1.7}$$

$$u|_{y=\delta} = u_e(x), \quad T|_{y=\delta_1} = T_e(x), \tag{1.8}$$

where x and y are the longitudinal and normal coordinates in a cartesian system attached to the convex flow surface; u and v are the projections of the velocity in the x and y directions, respectively; ρ is the density; p is the pressure; T is the temperature; M is the molecular weight of the gas; c_p is the specific heat of the gas; g is the acceleration of free fall; ν is the kinematic viscosity; δ and δ_1 are the dynamic and thermal boundary-layer thicknesses, respectively; μ and λ are the effective viscosity and thermal conductivity of the gas; k = 0, 1; α is the angle between the tangent to the surface and the horizontal plane; the indices M and T refer to the molecular and turbulent transport coefficients; and the indices w and e refer to the values of the functions on the flow surface (wall values) and at the external boundary of the boundary layer, respectively.

The system of equations (1.1)-(1.4) is deduced from the system of Navier–Stokes equations by the method of Prandtl [4, 8]. Unlike the classical boundary-layer equations, in the given formulation the equation of motion is invariant under projection onto the y axis in the form (1.3) due to the presence of the body force due to gravity. A pressure gradient in the x direction is initiated by the transverse pressure difference, and gradient flow takes place in the layer above the heated surface.

If we set ν_T and λ_T equal to zero and invoke the Boussinesq approximation, then for $\alpha = 0$ from (1.1)-(1.4) we obtain the equations used in [1, 3].

To determine u_e, T_e, and p_e it is necessary to solve the gasdynamical equations with appropriate boundary conditions. These equations cannot be solved analytically in the general case. However, for k = 0 in the case of isothermal and k = 1 in the case of isentropic flows it is possible to obtain integrals of these equations analytically:

$$\frac{u_e^2}{2} + \int \frac{dp_e}{\rho_e} + g\left(\int_0^x \sin\alpha\, dx + \delta\cos\alpha\right) = c_{1e}; \tag{1.9}$$

$$T_e = \mathrm{const}, \quad \frac{\rho_e}{p_e^{\varkappa}} = c_{2e}, \quad \varkappa = \frac{c_p}{c_V}, \tag{1.10}$$

where c_V is the specific heat at constant volume and c_{1e} and c_{2e} are constants, which in general change in transition from one streamline to another.

To determine the boundary-layer thicknesses we adopt the conditions

$$\left.\frac{\partial u}{\partial y}\right|_{y=\delta} = m_1, \quad \left.\frac{\partial T}{\partial y}\right|_{y=\delta_1} = m_2, \tag{1.11}$$

which generalize the standard conditions [5], where in general m_1 and m_2 are functions of x characterizing the vorticity of the flow and the temperature gradient at $y = \delta, \delta_1$. If the outer flow is isothermal, then $m_2 = 0$, whereas for adiabatic flows $m_2 = -g/c_p$. We have thus completed the statement of the problem, because once the velocity has been given, all other parameters of the outer flow can be determined.

2. Qualitative Analysis of the Problem on the Basis of the Momentum Integral Equation. If we transform to Dorodnitsyn–Howarth variables [8], eliminate v by means of (1.1), and change over to dimensionless notation in Eq. (1.2), we obtain the integrodifferential equation

$$u\frac{\partial u}{\partial x}+\left(v_w-\int_0^y\frac{\partial u}{\partial x}dy\right)\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}\left(\mu\rho\frac{\partial u}{\partial y}\right)-\frac{\sin\alpha}{\mathrm{Fr}}-\frac{1}{\rho}\frac{\partial p}{\partial x}-\frac{\partial p}{\partial y}\int_0^y\frac{1}{\rho}\frac{\partial\ln\rho}{\partial x}dy,$$

$$u=\frac{u'}{u_\infty},\ \rho=\frac{\rho'}{\rho_0},\ p=\frac{p'}{\rho_0u_\infty^2},\ \mathrm{Fr}=\frac{u_\infty^2}{gl},\ \mathrm{Re}=\frac{u_\infty\rho_0 l}{\mu_\infty}, \tag{2.1}$$

$$v_w=\frac{\rho_w'v_w'}{\rho_0u_\infty}\sqrt{\mathrm{Re}},\quad y=\frac{\sqrt{\mathrm{Re}}}{l}\int_0^{y'}\frac{\rho'}{\rho_0}dy',\ x=\frac{x'}{l},$$

where u_∞ and μ_∞ are the freestream velocity and viscosity, ρ_0 is the minimum density inside the boundary layer, l is a characteristic length, and the prime is attached to dimensioned quantities. Integrating (2.1) with respect to y from 0 to δ and making some simple transformations, we obtain an expression for the dimensionless frictional stress at the wall $\tau_w = \mu\rho\,\partial u/\partial y|_{y=0}$:

$$\tau_w=\tau_e-BU+\dot U\int_0^\delta(U-u)\,dy+\int_0^\delta[u(U-u)]\,dy-\left(U\dot U+\frac{\sin\alpha}{\mathrm{Fr}}\right)\times$$
$$\times\int_0^\delta\left(1-\frac{\rho_e}{\rho}\right)dy-\frac{\cos\alpha}{\mathrm{Fr}\sqrt{\mathrm{Re}}}\left[\int_0^\delta\frac{1}{\rho}\frac{\partial\ln\rho}{\partial x}dy\int_0^\delta\frac{\rho_e}{\rho}dy-\int_0^\delta\frac{(\delta-y)}{\rho}\frac{\partial\ln\rho}{\partial x}dy\right], \tag{2.2}$$
$$\dot U=\frac{dU}{dx},$$

where U is the dimensionless outer flow velocity. We analyze relation (2.2) from the point of view of vanishing of the quantity τ_w and the possibility of boundary-layer separation. We draw the following conclusions:

1. The outer-flow vorticity, characterized by the quantity $\tau_e = \mu\rho\,\partial u/\partial y|_{y=\delta}$, stimulates boundary-layer separation if $\tau_e < 0$ and inhibits separation if $\tau_e > 0$.

2. Injection always stimulates separation, whereas suction inhibits separation, because τ_w decreases with increasing $(\rho v)_w$, whereas for $(\rho v)_w < 0$ it increases.

3. The third term promotes a decrease of τ_w for $dU/dx < 0$ (i.e., for retarded flows) and inhibits a decrease for $dU/dx > 0$, since $U > u$ inside the boundary layer.

4. The fourth term has the form

$$\int_0^\delta\frac{\partial}{\partial x}[u(U-u)]\,dy=\int_0^\delta(U-u)\frac{\partial u}{\partial x}dy+\int_0^\delta u\left(\frac{\partial U}{\partial x}-\frac{\partial u}{\partial x}\right)dy.$$

It is seen that the first integral is positive for accelerated flows and negative for retarded flows. The second integral is negative if the acceleration inside the boundary layer is greater than the acceleration at the external boundary of the boundary layer.

5. If the freestream temperature is lower than the wall temperature, then $\rho_e/\rho > 1$. Therefore, the integral in the fifth term is negative. In other words, the fifth term decreases τ_w if the flow is retarded $(dU/dx < 0)$ and $\alpha < 0$.

6. The sixth term in relation (2.2) is associated with gravity forces. Inasmuch as $\rho_e/\rho \geqslant 1$, it is evident that

$$\left|\int_0^\delta\frac{1}{\rho}\frac{\partial\ln\rho}{\partial x}dy\int_0^\delta\frac{\rho_e}{\rho}dy\right|\geqslant\left|\delta\int_0^\delta\frac{1}{\rho}\frac{\partial\ln\rho}{\partial x}dy\right|\geqslant\left|\int_0^\delta\frac{(\delta-y)}{\rho}\frac{\partial\ln\rho}{\partial x}dy\right|.$$

Therefore, the sign of the bracketed expression in the last term of (2.2) is determined by the sign of $\int_0^\delta \frac{1}{\rho}\frac{\partial \ln \rho}{\partial x}dy$. The sign of this integral, in turn, depends on the sign of $\frac{\partial \ln \rho}{\partial x}$. Inasmuch as $\rho \sim 1/T_w$, we have $\frac{\partial \ln \rho}{\partial x} \sim -\frac{1}{T_w^2}\frac{dT_w}{dx}$, and separation can only occur under the condition that T_w decreases with increasing x. Conversely, with an increase in T_w we have $\partial \ln \rho/\partial x < 0$, so that τ_w increases and the probability of separation is diminished. It is important to note that in analyzing the sign of $\partial \ln \rho/\partial x$ in the general case it is necessary also to consider the sign of dT_e/dx. Specifically, if $T_e = T_e(x)$ and $T_w = T_w(x)$, then separation can occur if $dT_e/dx < 0$ and $dT_w/dx < 0$. Boundary-layer separation does not occur if $dT_e/dx > 0$ and $dT_w/dx > 0$. Finally, in the event of opposite signs for these two derivatives the sign of $\partial \ln \rho/\partial x$ depends on which derivative is the larger.

It thus follows from the analysis of relation (2.2) that in the space of the parameters Θ_w, B, Fr, Re, τ_e there can exist a surface separating the region corresponding to nonseparating flow from the part of the parameter space corresponding to separation flow.

3. Analytical Solution of the Problem by the Method of Shvets [5]. If flow is laminar and it is assumed that the product of the dimensionless forms of the viscosity, thermal conductivity, and density is equal to 1, the solution of the boundary-value problem (1.1)-(1.11) can be reduced to the integration of Eq. (2.1) for $\mu\rho = 1$ and the energy conservation equation

$$\frac{1}{\Pr}\frac{\partial^2\Theta}{\partial y^2} = u\frac{\partial\Theta}{\partial x} + \left(B - \int_0^y \frac{\partial u}{\partial x}dy\right)\frac{\partial\Theta}{\partial y},$$

in which $\Phi = T/T_\infty$, T_∞ = constant and the remaining variables are explained in Sec. 2.

Using the method of Shvets [5], we obtain the following in the second approximation for flow over a plate (U = 1, k = 0):

$$\frac{\Theta_2}{\Pr} = \frac{\dot{\Theta}_w y}{6\delta}(y^2-\delta_1^2) + \left(\frac{\dot{\Delta} y}{12\delta\delta_1} - \frac{\Delta\dot{\delta}_1 y}{12\delta\delta_1^2} + \frac{\Delta y}{24\delta_1\delta^2}\right)(y^3-\delta_1^3) + \frac{B\Delta y}{2\delta_1}(y-\delta) + \frac{\Theta_w}{\Pr} + \frac{\Delta y}{\Pr\delta_1},$$

$$u_2 = \frac{y}{\delta} - \frac{y(y^3-\delta^3)}{24\delta^3} + \frac{By(y-\delta)}{2\delta} + \gamma\Big\{\rho_e K[\beta I_3(\delta) - I_4(\delta)] \times$$
$$\times\left[I_2(y) - \frac{y}{\delta}I_2(\delta)\right] - \beta\left[I_1(y) - \frac{y}{\delta}I_1(\delta)\right] + I_5(y) - \frac{y}{\delta}I_5(\delta)\Big\},$$

$$\Delta = \Theta_e - \Theta_w,\ \gamma = \frac{1}{\mathrm{Fr}\sqrt{\mathrm{Re}}},\ p_e = p_\infty - \delta\gamma\left(\Theta_w + \frac{\Delta\delta}{2\delta_1}\right),\ \rho_e = \frac{p_e}{K(\Delta\delta/\delta_1 + \Theta_w)},$$

$$\beta = \gamma\rho_e I_0(\delta),\ I_0(\delta) = \frac{I_4(\delta)}{\gamma\rho_e I_3(\delta) - 1},\ I_1(y) = K\Big\{\frac{\Delta}{\delta_1\gamma^2}\Big\{\frac{1}{4\gamma^2}[[p_e + \gamma(\delta-y)]^2 \times$$
$$\times[2\ln(p_e + \gamma(\delta-y)) - 1] - (p_e+\gamma\delta)^2[2\ln(p_e+\gamma\delta) - 1]] -$$
$$-\frac{y^2}{2}[\ln(p_e+\gamma\delta)+1] + y\frac{p_e+\gamma\delta}{\gamma}\ln(p_e+\gamma\delta)\Big\} + \left[\frac{\Theta_w}{\gamma} + \frac{\Delta(p_e+\gamma\delta)}{\delta_1\gamma^2}\right] \times$$
$$\times\left[-\frac{y^2}{2(p_e+\gamma\delta)} + \frac{y}{\delta} + \frac{p_e+\gamma(\delta-y)}{\gamma^2}\ln\left(\frac{p_e+\gamma(\delta-y)}{p_e+\gamma\delta}\right)\right]\Big\},$$

$$I_2(y) = -\frac{\Delta y^2}{2\delta_1\gamma} + \left[\frac{\Theta_w}{\gamma} + \frac{\Delta(p_e+\gamma\delta)}{\delta_1\gamma^2}\right]\left[y - \frac{p_e+\gamma(\delta-y)}{\gamma}\ln\left(\frac{p_e+\gamma\delta}{p_e+\gamma(\delta-y)}\right)\right],$$

$$I_3(\delta) = \frac{K\delta}{p_e^2}\left(\Theta_w + \frac{\Delta\delta}{2\delta_1}\right),\ I_4(\delta) = \frac{K\delta}{2p_e}\left[2\dot{\Theta}_w + \frac{d}{dx}\left(\frac{\Delta}{\delta_1}\right)\right],\ I_5(y) = K\Big\{\frac{y^3}{6\gamma}\frac{d}{dx}\left(\frac{\Delta}{\delta_1}\right) +$$
$$+\left[\frac{\dot{\Theta}_w}{\gamma} + \frac{1}{\gamma^2}\frac{d}{dx}\left(\frac{\Delta}{\delta_1}\right)(p_e+\gamma\delta)\right]\Big[\frac{y^2}{2} - \frac{1}{4\gamma^2}[(p_e+\gamma(\delta-y))^2(2\ln(p_e +$$
$$+\gamma(\delta-y)) - 1) - (p_e+\gamma\delta)^2(2\ln(p_e+\gamma\delta)-1)] + \frac{y^2}{2}\times$$
$$\times\ln[p_e+\gamma(\delta-y)] - y\frac{p_e+\gamma\delta}{\gamma}\ln(p_e+\gamma\delta)\Big]\Big\}.$$

where the overhead dot signifies differentiation with respect to x.

To determine the thicknesses of the dynamic and thermal boundary layers we apply conditions (1.11), whereupon we obtain the ordinary differential equations

$$\dot{\delta}_i = F_i(\delta, \delta_1, \mathrm{Pr}, \mathrm{Fr}, \mathrm{Re}, \Theta_w, \Theta_e, U, K, B), \; \delta_i(0) = 0, \; i = 1, 2, \tag{3.1}$$

in which $B = v_w\sqrt{\mathrm{Re}}/u_\infty$, $K = RT_\infty/Mu_\infty^2$ are dimensionless parameters and for brevity the specific forms of the functions F_i are not given.

For $x \ll 1$ we have the asymptotic solution of the Cauchy problem (3.1) in the case of flow over a plate

$$\delta = 4\sqrt{x}, \; \delta_1 = 4\mathrm{Pr}^{-1/3}\sqrt{x}, \tag{3.2}$$

which agrees with the results of [5].

We can use (3.2) to obtain asymptotic expressions for the frictional stress and heat flux at the wall in dimensionless form for the case Θ_e = const, Θ_w = const, Pr = 1, U = 1:

$$\tau_w = \frac{1}{3\sqrt{x}} - \frac{B}{2} + \frac{5\gamma K(\Theta_w - 1)}{3p_e}; \tag{3.3}$$

$$q_w = \Delta\left(\frac{1}{3\sqrt{x}} - \frac{B}{2}\right). \tag{3.4}$$

An analysis of (3.3) indicates that free convection does not significantly affect the flow regime in a laminar layer under the conditions stipulated by the inequalities

$$1 \ll \mathrm{Re}_x < 10^5, \; 5\gamma K(\Theta_w - 1)/3p_e \ll 1/3\sqrt{x} - B/2,$$

in which Re_x is the instantaneous Reynolds number.

4. Results of Numerical Solution. Along with the analytical solution we have carried out a numerical integration of the boundary-value problem (1.1)-(1.11) for $\alpha = 0$, $m_1 = m_2 = 0$, corresponding to uniform flow over a horizontal plate with stable stratification of the medium [4].* Equations (1.1)-(1.4) are transformed in such a way that with the new variables [9] the domain of definition of Eqs. (1.2) and (1.3) varies from 0 to 1. We use a difference scheme constructed by the iteration-interpolation method described in [6, 7]. The scheme has first-order approximation in the y direction and first-order in the x direction. The program is tested by running a comparison with the numerical results of [3]. Table 1 gives the values of the dimensionless frictional stress $a = F''(\xi, 0)$ and the dimensionless heat flux $b = -\Theta'(\xi, 0)$ (in the notation of [3]) in the Boussinesq approximation, along with the same values obtained by solution of the boundary-value problem (1.1)-(1.11). Here, Gr_x and Re_x are the instantaneous Grashof and Reynolds numbers [3], $c = (T_w - T_e)/T_w = 0.06$, and the subscript 1 is attached to the quantities obtained by numerical integration of the boundary-value problem (1.1)-(1.11). It is seen that the agreement with the numerical results of [3] is observed. It is curious that the agreement deteriorates with increasing value of c. Thus, for $\mathrm{Gr}_x/\mathrm{Re}_x^{5/2} = 0.4$ and $c = 0.25$ the quantity a_1 exceeds a by 5%, and b_1 exceeds b by 3%, whereas for $c = 0.77$ the error of a increases to 50%, and that of b to 13%. Consequently, the Boussinesq approximation, as expected, yields a sizable error in the determination of the gross characteristics of the boundary layer.

It has been determined as a result of the numerical calculations that for $0 < x \quad 1$, $0 < B < 2.5$, $1 < \Theta_w \leqslant 5$ the error of the asymptotic expressions (3.3) and (3.4) for an isothermal plate is not greater than 20%.

To test the results of the qualitative analysis we have carried out numerical calculations for

*The numerical calculations are performed with regard for the temperature dependence of the quantities c_p, μ_M, λ_M for air [10].

TABLE 1

$Gr_x/Re_x^{5/2}$	0	0,2	0,4	0,6	0,8	1,0
a	0,33206	0,58915	0,77849	0,93694	1,07592	1,20569
a_1	0,33404	0,58171	0,76982	0,92901	1,06927	1,18844
b	0,29268	0,33751	0,36178	0,37949	0,39423	0,40658
b_1	0,29363	0,33441	0,35928	0,37928	0,39264	0,40461

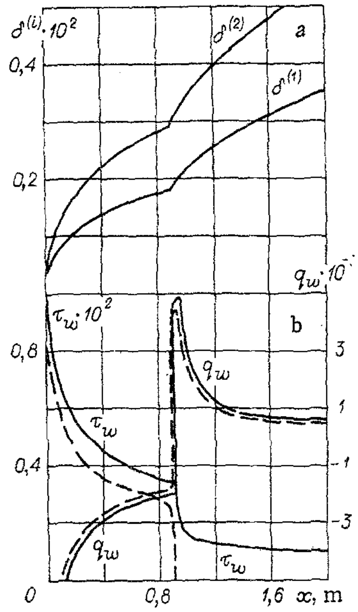

Fig. 1

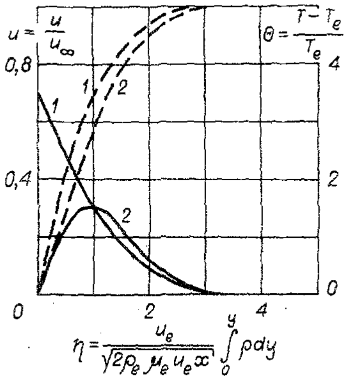

Fig. 2

$$T_w=\begin{cases}T_1, & 0<x<x_1,\\ T_e-(T_e-T_1)\exp[-3000(x-x_1)^2] & \text{for}\quad x\geqslant x_1\,;\end{cases} \tag{4.1}$$

$$(\rho v)_w=\begin{cases}(\rho v)_w=\text{const} & \text{for}\quad 0<x<x_2,\\ (\rho v)_{w1}\exp[-3000(x-x_2)^2] & \text{for}\quad x\geqslant x_2.\end{cases} \tag{4.2}$$

The momentum-loss thickness $\delta^{(1)}$, the energy-loss thickness $\delta^{(2)}$, τ_w, and q_w for laminar flow with parameters $(\rho v)_w = 0$, $x_1 = 0.9$ m, $u_e = 1$ m/sec, $T_1 = 1300°K$, and $T_e = 300°K$ are given in Fig. 1a, b in the SI system. The dashed curves represent the results obtained by the method of Shvets [5].* It follows from an analysis of the curves in Figs. 1a and 1b that in the interval of abrupt temperature drop the momentum-loss thickness $\delta^{(1)}$ and energy-loss thickness $\delta^{(2)}$ as well as the frictional stress and heat flux vary abruptly. The sharp reduction in the frictional stress at the wall, beginning with $x = x_1$, is explained by the fact that the positive temperature gradient increases rapidly in the interval of abrupt temperature drop, thereby retarding the flow and, accordingly, diminishing the frictional stress. It is well known that the boundary layer overflows in the case of flow separation. This fact accounts for the abrupt increase in the boundary-layer thickness and the momentum- and energy-loss thicknesses. Thus, for $x > x_1$ the boundary layer enters the presentation state. The change of sign of q_w is explained by the formation of a thermal curtain for $x > x_1$. This conclusion follows from a comparison of the temperature profiles (solid curves 1 and 2) and velocity profiles (dashed curves 1 and 2) for $x = 0.9x_1$ and $x = 1.1x_1$ in Fig. 2. It is seen that the temperature profile for $x > x_1$ represents a nonmonotonic function, while the velocity profile tends to contract.

*The system of equations (3.4) is solved numerically in this case by the Runge–Kutta method.

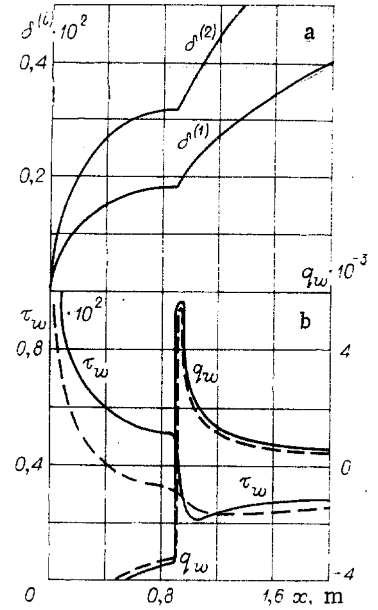

Fig. 3

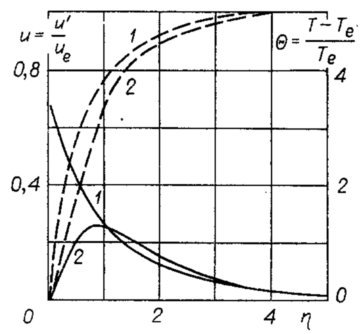

Fig. 4

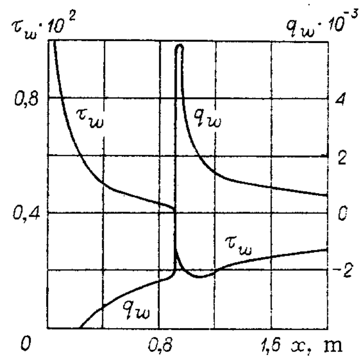

Fig. 5

The disparity between the results obtained by the method of Shvets [5] and by numerical integration is attributable to the fact that for $x > x_1$ the temperature profile undergoes a drastic change, which is not adequately approximated within the framework of the second approximation of the method of Shvets [5].

Figure 3a, b gives the quantities $\delta^{(1)}$, $\delta^{(2)}$, τ_w, q_w in the SI system for turbulent flow (solid curves) with parameters $(\rho v)_w = 0$, $x_1 = 0.9$ m, $u_e = 1$ m/sec, $T_w = 1300°K$, and $T_e = 300°K$, while Fig. 4 gives the temperature (solid curves 1 and 2) and velocity (dashed curves 1 and 2) profiles for $x = 0.9x_1$ and $x = 1.1x_1$. It is seen that, by contrast with the laminar case, for turbulent flow there is more rapid growth of the momentum- and energy-loss thicknesses $\delta^{(1)}$ and $\delta^{(2)}$, the velocity profiles are tighter, and the wall temperature gradient increases. The dashed curves in Fig. 3b correspond to the case $\partial p/\partial y = 0$ and, hence, $\partial p/\partial x = 0$, as is typical of forced convection. It follows from an analysis of the graphs that the presence of gravity forces significantly affects the friction at the surface. As for the heat flux, the dashed and solid curves differ only very slightly, i.e., the heat flux is weakly dependent on the force of gravity. At $x = x_1$ there is an abrupt change in the heat flux, which changes sign, because for $0 < x < x_1$ heat is withdrawn from the wall by the cold freestream flow, while for $x > x_1$ the opposite pattern is observed, i.e., the gas flow imparts heat to the cold wall (see Fig. 4). The results of a calculation of a turbulent boundary layer with regard for body forces and injection are given in Fig. 5 in the SI system. For $x < x_2$ the mass flow rate on the surface of a plate is $(\rho v)_w = 2.33 \cdot 10^{-3}$ kg/m²sec, while for $x > x_2$ the quantity $(\rho v)_w$ decreases in accordance with (4.2). The length of the constant-injection zone is $x_2 = 0.9$ m, and all other characteristics are taken the same as in the version represented by Fig. 3a, b. According to the conclusions drawn from an analysis of the integral equation (2.2), the frictional stress and heat flux for this version are smaller than in the noninjection case.

Thus, with the aid of computer calculations we have corroborated the predictions as to the influence of injection and stratification inside the boundary layer on the value of τ_w. It must be emphasized that sufficient conditions for the existence of separation do not follow from the qualitative analysis of relation (2.2), whereas the presence of negative terms on the right-hand side of (2.2) is a necessary condition for $\tau_w = 0$ to hold. The boundary-layer equations, as we know, do not describe separation flows. The latter are described within the framework of the Navier–Stokes equations. On the other hand, the pre-separation state of the boundary layer can be deduced by numerical calculations (see Figs. 1, 3, and 5).

The foregoing results augment in some measure the well-known theory of the thermal curtain [11] and should prove useful for the analysis of heat and mass transfer associated with forest fires [12].

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